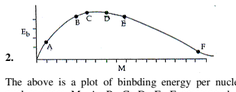
AIEEE 2009 ISET A1

This question contains statements-1 and statement-2 .of the four choices given after the statements, choose the one that best describes the two statements.
(1) Statement -1 - is true, Statement-2 is false.
(2) Statement -1 is true, Statement - 2 is true, Statement -2 is the correct explanation of Statement-1
(3) Statement -1 is true, Statement -2 is true, Statement-2 is not the correct explanation of Statement-1.
(4) Statement-1 is false, Statement-2 is true.

1. Statement 1 : For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.
Statement 2 : The net work done by a conservative force on an object moving along a closed loop is zero.
Ans (3)

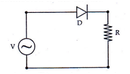
2.



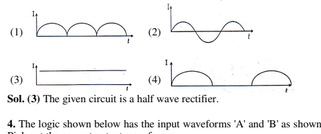
The above is a plot of binbding energy per nucleon E_b, against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei. Consider four reactions :
(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$
(iii) $D + E \rightarrow F + \varepsilon$ and (iv) $F \rightarrow D + E + \varepsilon$
(1) (i) and (iv) (2) (i) and (iii) (3) (ii) and (iv) d) (ii) and (iii)

Sol. : (1) All reactions which proceed in the direction of increasing B.E. per neucleon will release energy. Hence (I) and (IV) will be true.

3. A *p-n* (D) shown in the figure can act as rectifire. An alternating current source (V) is connected in the circuit.

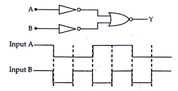


The current (I) in the resistor (R) can be shown by :

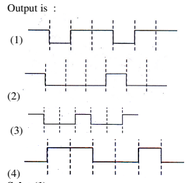


Sol. (3) The given circuit is a half wave rectifier.

4. The logic shown below has the input waveforms 'A' and 'B' as shown. Pick out the correct output waveform.



Output is :



Sol. : (1)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

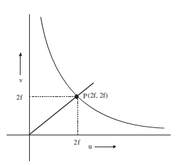
Truth table of AND gate

5. If *x*, *v* and *a* denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period *T*, then, which of the following does not change with time ?
(1) $a^2T^2 + 4\pi^2v^2$ (2) aT / x
(3) $aT + 2\pi v$ (4) aT / v

Sol. : (2) $\frac{aT}{x} = \frac{\omega^2 xT}{x} = \omega^2 T$ = constant

6. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45^0 with the x-axis meets the experiment curve at *P*. The cordinates of *P* will be :
(1) $(2f, 2f)$ (2) $\left(\frac{f}{2}, \frac{f}{2}\right)$ (3) (f, f) (4) $(4f, 4f)$

Sol. : (1)



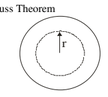
7. An thin uniform rod of length *l* and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω . Its centre of mass rises to a maximum height of :
(1) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (2) $\frac{1}{6}\frac{l\omega}{g}$ (3) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (4) $\frac{1}{6}\frac{l^2\omega^2}{g}$

Sol. : (4) By conservation of mechanical energy

$\frac{1}{2}I\omega^2 = mgh_{com}$; $\frac{1}{2}\frac{ml^2}{3}\omega^2 = mgh_{com}$; $h_{com} = \frac{l^2\omega^2}{6g}$

8. Let P (r) = $\frac{Q}{\pi R^4}r$ be the charge density distribution for a soilid of radius R and total change Q. For a point 'p' inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is :
(1) 0 (2) $\frac{Q}{4\pi\epsilon_0 r_1^2}$ (3) $\frac{Qr_1^2}{4\pi\epsilon_0 R^4}$ (4) $\frac{Qr_1^2}{3\pi\epsilon_0 R^4}$

Sol. : (3) Applying Gauss Theorem



$\oint E \cdot ds \cdot \cos\theta = \frac{Q_{in}}{\epsilon_0}$

$E \cdot 4\pi r_1^2 \cdot 1 = \left[\frac{\int_0^r \frac{Q}{\pi R^4} \cdot r \cdot 4\pi r^2 \cdot dr}{\epsilon_0}\right]$; $E = \frac{Qr_1^2}{4\pi\epsilon_0 R^4}$

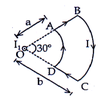
9. The transition from the state n=4 to n=3 in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from :
(1) $2 \rightarrow 1$ (2) $3 \rightarrow 2$ (3) $4 \rightarrow 2$ (4) $5 \rightarrow 4$
Sol. : (4) As we go to higher energy levels difference of energy between two consecutive levels decreses, so wavelength increases, $5 \rightarrow 4$

10. One kg of a diatomic gas is at a pressure of $8\times10^4\,N/m^2$. The density of the gas is $4kg/m^3$. What is the energy of the gas due to its thermal motion?
(1) 3×10^4 J (2) 5×10^4 J (3) 6×10^4 J (4) 7×10^4 J

Sol. : **(2)** $U = \frac{f}{2}PV = \frac{5}{2}\times8\times10^4\times\frac{1}{4} = 5\times10^4$ J

This question contains statements-1 and statement-2 of the four choices given after the statements, choose the one that best describes the two statements.
11. Statement 1 : The temperature dependence of resistance is usually given as $R = R_0(1 + \alpha\Delta t)$. The reisstance of a wire changes from 100 Ω to 150 Ω when its temperature is increased from 27^0C to 227^0 C. This implies that $\alpha = 2.5\times10^{-3}/^0$ C
Statement 2 : $R = R_0(1 + \alpha\Delta t)$ is valid only when the change in the temperature ΔT is small and $\Delta R = (R - R_0) << R_0$
Sol. : (4)
Basic concept hence statement 1 is false and 2 is true

Directions : Question numbers 12 and 13 are based on the following paragraph.
A current loop ABCD is held fixed on the plane of the paper as shown in the figure. "The arcs BC (radius=b) and DA (radius=a) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30^0. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin.



12. The magnitude of the magnetic field (B) due to the loop ABCD at the origin (O) is :
(1) Zero (2) $\frac{\mu_0 I(b-a)}{24ab}$
(3) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$ (4) $\frac{\mu_0 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

Sol. : (2) $B_{net} = \frac{\mu_0}{4\pi}\frac{i}{a}\frac{\pi}{6} - \frac{\mu_0}{4\pi}\frac{i}{b}\frac{\pi}{6}$; $\frac{\mu_0}{24ab}(b-a)$

13. Due to the presence of the current I_1 at the origin :
(1) The forces on AB and DC are zero
(2) The forces on AD and BC are zero.
(3) The magnitude of the net force on the loop is given by $\frac{I_1 I}{4\pi}\mu_0\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$
(4) The magnitude of the net force on the loop is given by $\frac{\mu_0 I_1}{24ab}(b-a)$

Sol. : (2) $\vec{B}$ and $\vec{dl}$ are parallel at every point so Force on AD & BC is zero.

14. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From the data, the wavelength of the unknown light is :
(1) 393.4 nm (2) 885.0 nm (3) 442.5 nm (4) 776.8 nm
Sol. : (3) Position of n^{th} bright fringe frrom central maxima is Given by

$Y_n = \frac{n\lambda D}{d}$; Now, $Y_3 = Y_4 \Rightarrow \lambda = 442.5$nm

15. Two points P and Q are maintained at the potentials of 10V and- 4V, respectively. The work done in moving 100 electrons from P to Q is :
(1) -9.60×10^{-17} J (2) 9.60×10^{-17} J
(3) -2.24×10^{-16} J (4) 2.24×10^{-16} J
Sol. : (4) Work done = $q[V_f - V_i] = 2.24\times10^{-16}$ J

16. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68eV. The work function of the metal is : (hc = 1240 eV.nm)
(1) 3.09 eV (2) 1.41 eV (3) 1.51 eV (4) 1.68 eV
Sol. : (2) $\phi = E - K$ **= 1.42 eV**

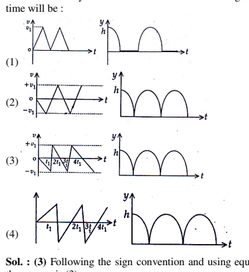
17. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 *s* is :
(1) 10 units (2) $7\sqrt{2}$ units (3) 7 units (4) 8.5 units

Sol. : (2) $u = 3\hat{i} + 4\hat{j}$; $a = 0.4\hat{i} + 0.3\hat{j}$
$v_x(t) = u_x + at = 7$ $v_y(t) = u_y + at = 7$
$v = \sqrt{v_x^2 + v_y^2} = 7\sqrt{2}$

18. A motor cycle starts rom rest and accelerates along a straight path at $2m/s^2$. At the starting point of the motor cycle there is a stationery electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle wast at rest? (Speed of sound = 330 ms^{-1})
(1) 49 m (2) 98 m (3) 147 m (4) 196 m

Sol.: (2) $v' = \left[\frac{u - u_0}{u}\right]v$; $u_0 = 19.8 m/sec$; $v^2 = u^2 + 2as \Rightarrow s = 98.01$m

19. Consider a rubber ball freely falling from a height h = 4.9 m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic.
Then the velocity as a function of time and the height as a function of time will be :



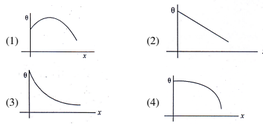
Sol. : (3) Following the sign convention and using equations of motion the answer is (3)

20. A charge Q is placed at each of the opposite cornes of square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals :
(1) $-2\sqrt{2}$ (2) -1 (3) 1 (4) $-\frac{1}{\sqrt{2}}$

Sol. : (1) As $\vec{F_1} + \vec{F_2} + \vec{F_3} = \vec{0} \Rightarrow \sqrt{2}\frac{kqQ}{a^2} = \frac{KQ^2}{2a^2}$

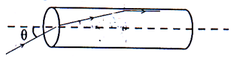
$\Rightarrow \frac{Q}{q} = -2\sqrt{2}$ (-ve as sign of the charges must be opposite)

21. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures ?



Sol. : (2) -ve temperature gradient is constant in the steady state.

22. A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is sorrounded by air. A light ray is incident at the mid point of one end of the rod as shown in the figure.



The incident angle θ for which of the light rays grazes along the wall of the rod is :
(1) $\text{Sin}^{-1}\left(1/2\right)$ (2) $\text{Sin}^{-1}\left(\sqrt{3}/2\right)$
(3) $\text{Sin}^{-1}\left(2/\sqrt{3}\right)$ (4) $\text{Sin}^{-1}\left(1/\sqrt{3}\right)$

Sol. : (4) r = 90^0 - C $\mu = \frac{1}{\sin C}$
$\Rightarrow r = 30^0$ $\Rightarrow C = 60^0$ as $\mu = \frac{\sin i}{\sin r}$
$\frac{2}{\sqrt{3}} = \frac{\sin\theta}{\sin 30^0} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

23. Three sound waves of equal amplitudes have frequencies (ν -1), ν , (ν +1). They superpose to give beats. The number of beats produced per second will be :
(1) 4 (2) 3 (3) 2 (4) 1
Sol. : (4) Considering local maxima 2 beats will be produced while if we ignore local maxima 1 beat will be produced.

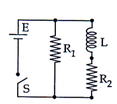
24. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g= the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is :
(1) 2R (2) $\frac{R}{\sqrt{2}}$ (3) R/2 (4) $\sqrt{2}$R

Sol. : (1) $g(h) = g_0\left(\frac{R}{R+h}\right)^2 \Rightarrow h = 2R$

25. The wires are made of the same material and have the same volume. However wire 1 has cross- sectional area. A and wire 2 has cross-sectional area 3 A. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?
(1) F (2) 4F (3) 6F (4) 9F
Sol. : (4) $Y = \frac{F \cdot L}{A \cdot \Delta x} = \frac{F \cdot V}{A^2 \Delta x}$; $F \propto A^2 \Rightarrow F_2 = 9F$

26. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (=0.5^0), then the least count of the instrument is :
(1) one minute (2) half minute (3) one degree (4) half degree
Sol. : (1) L.C.= $\frac{\text{Value of one part on main scale}}{\text{Total no of divisions}} = \left[\left(\frac{.5}{30}\right)^0 \times 60\right] = 1'$

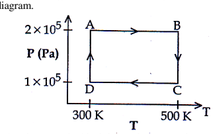
27.



An inductor of inductane L = 400 mH and resistors of resistances R_1 = 2 Ω and R_2=2 Ω are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at t=0. The potential drop across L as a function of time is :
(1) $6e^{-5t}$ V (b) $\frac{12}{t}e^{5t}$ V (3) $6\left(1 - e^{-t/0.2}\right)$ V (4) $12e^{-5t}$ V

Sol. : (4) $i(t) = I_0\left(1 - e^{-Rt/L}\right) \Rightarrow L \cdot \frac{di}{dt} = 12e^{-5t}$

Directions : Question numbers 28, 29 and 30 based on the follwoing paragraph.
Two moles of helium gas are taken over the cycle ABCDA, as shown in the P-T diagram.

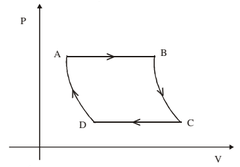


28. Assuming the gas to be ideal the work done on the gas in taking from A to B is :
(1) 200 R (2) 300 R (3) 400 R (4) 500R
Sol. (3)

29. The work done on the gas in taking it from D to A is :
(1) -414 R (2) +414 R (3) -690 R (4) + 690 R
Sol. (1)

30. The net work done on the gas in the cycle ABCDA is :
(1) Zero (2) 276 R (3) 1076 R (4) 1904 R
Sol. : (2) For Questions 28, 29 and 30

A $\rightarrow$ B Isobaric Expansion
B $\rightarrow$ C Isothermal Expansion
C $\rightarrow$ D Isobaric Compression
D $\rightarrow$ A Isothermal Compression



Work done = $\int P \cdot \Delta V$
A $\rightarrow$ B = $\mu R [T_f - T_i] = 400R$
D $\rightarrow$ A = $\mu RT \ln\left[\frac{P_f}{P_i}\right]$
So **Ans (28)** work done = 400 R
(29) - 414 R and **(30)** 276 R

31. Knowing that the Chemistry of lanthanoids (Ln) is dominated by its + 3 oxidation state, which of the following statements is incorrect ?
(1) Because of the large size of the Ln (III) ions the bonding in its compounds is predominently ionic in character
(2) The ionic sizes of Ln (III) decrease in general with increasing atomic number
(3) Ln (III) compounds are generally colourless
(4) Ln (III) hydroxides are mainly basic in character
Sol. : [3]

32. A liquid was mixed with ethanol and a drop of concentrated H_2SO_4 was added. A compound with a fruity smell was formed. The liquid was :
(1) CH_3OH (2) HCHO (3) CH_3COCH_3 (4) CH_3COOH

Sol. : [4] $A + C_2H_5OH \xrightarrow{H^+}$ Ester
(Fruity smell)
i.e. $A \rightarrow CH_3COOH$

33. Arrange the carbanions, $(CH_3)_3\bar{C}, \bar{C}Cl_3, (CH_3)_2\bar{C}H, C_6H_5\bar{C}H_2$ in order of their decreasing stability :
(1) $C_6H_5\bar{C}H_2 > \bar{C}Cl_3 > (CH_3)_3\bar{C} > (CH_3)_2\bar{C}H$
(2) $(CH_3)_2\bar{C}H > \bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_3\bar{C}$
(3) $\bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_2\bar{C}H > (CH_3)_3\bar{C}$
(4) $(CH_3)_3\bar{C} > (CH_3)_2\bar{C}H > C_6H_5\bar{C}H_2 > \bar{C}Cl_3$
Sol. : [3]

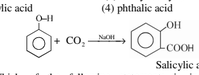
$\bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_2\bar{C}H > (CH_3)_3\bar{C}$
$d\pi - p\pi$ Resonance
Stabilization
As alkyl group $\uparrow$ stability of C $\downarrow$

34. The alkene that exhibits geometrical isomerism is :
(1) propene (2) 2 - methyl propene
(3) 2 - butene (4) 2 - methyl -2- butene
Sol.: [3] $CH_3 - CH = CH - CH_3$

35. In which of the following arrangements the sequence is not strictly according to the property written against it ?
(1) $CO_2 > SiO_2 < SnO_2 < PbO_2$: increasing oxidising power
(2) HF > HCl < HBr < HI : increasing acid strength
(3) $NH_3 > PH_3 < AsH_3 < SbH_3$: increasing basic strength
(4) B > C < O < N : increasing first ionizaation enthalpy
Sol. [3] The correct basic strength of 15^{th} group hydrides is
$NH_3 > PH_3 > AsH_3 < SbH_3$
The basic strength of hydrides decreases down the group

36. The major product obtained on interaction of phenol with sodium hydroxide and carbon dioxide is:
(1) benezoic acid (2) salicyladehyde
(3) salicylic acid (4) phthalic acid

Sol.: [3]



Salicylic Acid

37. Which of the following statements is incorrect regarding physisssorptions ?
(1) It occurs because of van der Waal's forces
(2) More easily liquefiable gases are absorbed readily
(3) Under high pressure it results into multi molecular layer on adsorbent surface
(4) Enthalpy of adsorption ($\Delta H_{adsorption}$) is low and positive
Sol. [4] Enthalpy of adsorption is negative

38. Which of the following on heating with aqueous KOH, produces acetaldehyde ?
(1) CH_3COCl (2) CH_3CH_2Cl
(3) CH_3ClCH_2Cl (4) CH_3CHCl_2
Sol. [4] $CH_3CHCl_2 \xrightarrow{aq.KOH} CH_3 - CH = O$

39. In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%. Certainity with which the position of the electron can be located is ($h = 6.6\times10^{-34}\,kg\,m^2s^{-1}$, mass of electron, $e_m = 9.1\times10^{-31}$kg) :
(1) 1.52×10^{-4}m (2) 5.10×10^{-3}m
(3) 1.92×10^{-3}m (4) 3.84×10^{-3}m

Sol. [3] $\Delta V = 600\times\frac{0.005}{100} = 0.030$

$\Delta x.\Delta p = \frac{h}{4\pi}$ (Heisenberg uncertainty principle)

$\Delta x.m\Delta v = \frac{h}{4\pi}$

$\Delta x.9.1\times10^{-31}\times3\times10^{-2} = \frac{6.6\times10^{-34}}{4\times3.14}$

$\Delta x = 1.92\times10^{-3}$ m

40. In a fuel cell methanol is used as fuel and oxygen gas is used as an oxidizer. The readction is :

$CH_3OH(l) + 3/2\,O_2(g) \longrightarrow CO_2(g) + 2H_2O(l)$ At 298 K standard Gibb's energies of formation for $CH_3OH(l), H_2O(l)$ and CO_2 (g) are – 166.2, –237.2 and –394.4 kJ mol^{-1} respectively. If standard enthalpy of combustion of methanol is – 726 kJ mol^{-1}, efficiency of the fuel cell will be :
(1) 80% (2) 87% (3) 90% (4) 97%

Sol. $CH_3OH + \frac{3}{2}O_2 \longrightarrow CO_2 + 2H_2O$

$\Delta G^\circ = \sum(G_f^\circ)_{product} - \sum(G_f^\circ)_{Reactant}$

$\left[\Delta G_f^\circ[CO_2] + 2\times\Delta(G_f^\circ)_{H_2O}\right] - \left[\Delta G^\circ_{f(CH_3OH)} + \frac{3}{2}\times\Delta G^\circ_{f(O_2)}\right]$ ($\Delta G^\circ_{f(O_2)} = 0$)

$= \left[(-394.4) + 2\times(-237.2)\right] - \left[(-166.2)\right]$

$= -703.60$

$\Delta H_c^\circ = -726.00$

% efficiency = $\left(\frac{\Delta G_c^\circ}{\Delta H_c^\circ}\right)\times100$

$= \frac{(-702.60)}{(-726)}\times100$

$= 96.78\% = 97\%$

41. Two liquids X and Y form an ideal solution. At 300 K, vapour pressure of the solution containing 1 mol of Y and 3 mol of Y is 550 mmHg. At the same temperature, if 1 mol of Y is further added to this solution, vapour pressure of the solution increases by 10mmHg. Vapour pressure (in mmHg) of X and Y in their pure states will be respectively :
(1) 200 and 300 (2) 300 and 400
(3) 400 and 600 (4) 500 and 600

Sol. (3) 1mol x p_x^0
3mol y p_y^0

$P_s = 550$mmHg

$550 = \left(\frac{1}{4}\right)p_x^0 + \left(\frac{3}{4}\right)p_y^0$.....(1)

1mol x p_x^0
4mol y p_y^0

$P_s = 560$mmHg

$560 = \left(\frac{1}{5}\right)p_x^0 + \left(\frac{4}{5}\right)p_y^0$.....(2)

$\left(eq.(1)\times\frac{4}{5}\right) - eq(2)$

$\frac{4}{5}\times550 = \frac{1}{5}p_x^0 + \frac{3}{5}p_y^0$

$560 = \frac{1}{5}p_x^0 + \frac{4}{5}p_y^0$
– – –

$120 = \frac{1}{5}p_y^0$

$p_y^0 = 600$

from eq. (2)

$p_x^0 = 400$

42. The half life period of a first order chemical reaction is 6.93 minutes. The time required for the completion of 99% of the chemical reaction will be (log 2 = 0.301) :
(1) 230.3 minutes (2) 23.03 minutes
(3) 46.06 minutes (4) 460.6 minutes

Sol. [3] $t_{\frac{1}{2}} = 6.93$ min

$t = \frac{2.303}{k}\log\left(\frac{C_0}{C}\right)$

$C = C_0 - .099C_0 = 0.01C_0$

$t = \frac{2.303}{k}\log\left(\frac{C_0}{0.01C_0}\right)$

$= \frac{2.303}{k}\log100$

$= \frac{2.303}{k}\times2$

$= \frac{2.303}{0.693}\times t_{\frac{1}{2}}\times2\left(t_{\frac{1}{2}} = \frac{0.693}{k}\right)$

$= \frac{2.303}{0.693}\times6.93\times2$

$= 46.06$ min

43. Given :

$E^0_{Fe^{3+}/Fe} = -0.036\,V, E^0_{Sn^{2+}/Fe} = -0.439\,V$

The value of standard electric potenital for the charge, $Fe^{3+}_{(aq)} + e^- \rightarrow Fe^{2+}(aq)$ will be :

(1) 0.072 V (2) 0.385 V (3) 0.770 V (4) – 0.270 V

Sol. [3] $Fe^{+++} + 3e^- \longrightarrow Fe \quad E^\circ = -0.036$

$$\frac{Fe \longrightarrow Fe^{++} + 2e^- \quad E^\circ = +0.439}{Fe^{+++} + e^- \longrightarrow Fe^{++} \quad \Delta G = \Delta G_1 + \Delta G_2}$$

$$-1\times F\times E = \left[-3\times F\times(-0.036)\right] + \left[-2\times F\times 0.439\right] \qquad E = 0.77V$$

44. On the basis of the following thermochemical data : $(\Delta G^\circ f\, H^+_{(aq)} = 0)$

$$H_2O(l) \longrightarrow H^+(aq) + OH^-(aq); \Delta H = 57.32 kj$$

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l); \Delta H = 286.20 \ \ kj$$

The value of enthalpy of formation of CH^- ion at 25°C is :

(1) –22.88kj (2) –228.88kj (3) +228.88kj (4) –343.52kj

Sol. [2] For enthalpy of formation of OH^-

$$\frac{1}{2}O_2(g) + \frac{1}{2}H_2(g) \longrightarrow OH^- \quad \Delta H = ?$$

given

(1) $H_2O(l) \longrightarrow H^+(aq.) + OH^-(aq.) \Delta H = 57.32kJ$

(2) $H_2(g) + \frac{1}{2}O_2(g) = H_2O(l) \Delta H = -286.20kj$

(3) $\frac{1}{2}H_2(g) \longrightarrow H^+ \Delta H = 0$

eq. (1) + eq. (2) + reverse of eq. (3)

$$H_2(g) + \frac{1}{2}O_2(g) + H_2O(l) + H^+ \longrightarrow H^+ + OH^- + H_2O(l) + \frac{1}{2}H_2O$$

$$\frac{1}{2}H_2 + \frac{1}{2}O_2 \longrightarrow OH^-$$

$$\Delta H = (57.32) + (-286.70)$$

$$\Delta H = -228.88kj$$

45. Copper crystallises in fcc with a unit cell length of 361 pm. What is the radius of copper atom ?

(1) 108 pm (2) 127 pm (3) 157 pm (4) 181 pm

Sol. [2] In fcc $r = \frac{a}{2\sqrt{2}}$

$a = 361pm$

$r = \frac{361}{2\sqrt{2}} = 127 \ pm$

46. Which of the following has an optical isomer ?

(1) $[CO(NH_3)_5Cl]^+$ (2) $[CO(en)(NH_3)_2]^{2+}$

(3) $[CO(H_2O)_4(en)]^{3+}$ (4) $[CO(en)_2(NH_3)_2]^{3+}$

Sol. (4) Octahedral complex $\left[Co(en)_2(NH_3)_2\right]^{+3}$ exist in 2 geometrical isomeric forms and cis isomer can show optical isomerism.

47. Solid $Ba(NO_3)_2$ is gradually dissolved in a 1.0×10^{-4} M Na_2CO_3 solution. At what concentration of Ba^{2+} will a precipitate begin to form ? (K_{sp} for Ba $CO_3 = 5.1\times10^{-9}$) :

(1) 4.1×10^{-5} M (2) 5.1×10^{-5} M

(3) 8.1×10^{-8} M (4) 8.1×10^{-7} M

Sol. [2] $Ba(CO_3)_2 \longrightarrow Ba^{++} + CO^{-2}$

$$k_{sp} = \left[Ba^{++}\right]\left[CO_3^{-2}\right]$$

$$5.1\times10^{-9} = \left[Ba^{++}\right]\times\left[1\times10^{-4}\right]$$

$$\left[Ba^{++} = \frac{5.1\times10^{-9}}{1\times10^{-4}} = 5.1\times10^{-5}\right]$$

48. Which one of the following reactions of Xenon compounds is not feasible ?

(1) $XeO_3 + 6HF \rightarrow XeF_6 + 3H_2O$

(2) $3XeF_6 + 6H_2O \rightarrow 2Xe + XeO_3 + 12HF + 1.5O_2$

(3) $2XeF_2 + 2H_2O \rightarrow 2Xe + 4HF + O_2$

(4) $XeF_6 + RbF \rightarrow Rb[XeF_7]$

Sol. (1) $XeF_6 + 3H_2O \rightarrow XeO_3 + 6HF$ (its spontaneous)

49. Using MO theory predict which of the following species has the shortest bond length ?

(1) O_2^{2+} (2) O_2^+ (3) O_2^- (4) O_2^{2-}

Sol. [1]

Sol. Bond order $= \frac{1}{2}[10-4] = 3$

Bond order $\alpha \frac{1}{\text{Bond length}}$

50. In context with the transition elements, which of the following statements is incorrect ?

(1) In addition to the normal oxidation states, the zero oxidation state is also shown by these elements in complexes

(2) In the highest oxidation states, the transition metal show basic character and form cationic complexes

(3) In the highest oxidation states of the first five transition elements (Sc to Mn), all the 4s and 3d electrons are used for bonding

(4) Once the d^5 configuration is exceeded, the tendency to involve all the 3d electrons in bonding decreases

Sol. [2] In the higher oxidation state, the transition metal shows acidic character and forms anionic complexes.

51. Calculate the wavelength (in nanometer associated with a proton moving at $1.0\times10^3 ms^{-1}$ (Mass of proton $= 1.67\times10^{-27}$kg and $h = 6.63\times10^{-34}$ Js):

(1) 0.032 nm (2) 0.40 nm (3) 2.5 nm (4) 14.0 nm

Sol.: [2] $\lambda = \frac{h}{mv} = \frac{6.03\times10^{-34}}{1.67\times10^{-27}\times10^3}$ m

3.97×10^{-10} m

$= 3.97\times10^{-10}$ m $= 3.97\times10^{-10}\times10^9$

$= 0.397$nm

$= 0.4$nm

52. A binary liquid solution is prepared by mixing n-heptane and ethanol. Which one of the following statements is correct regarding the behaviour of the solution ?

(1) The solution formed is an ideal solution

(2) The solution is non-ideal, showing +ve deviation from Raoult's Law

(3) The solution is non-ideal, showi8ng –ve deviation from Raoult's Law

(4) n-heptane shows +ve deviation while ethanol shows –ve deviation from Raoult's Law

Sol. [2] normal heptane decreases interaction between ethanol molecules so they show positive deviation from Raoult's Law

53. The number of stereoisomers possible for a compound of the molecular formula $CH_3 - CH = CH - CH(OH) - Me$ is :

(1) 3 (2) 2 (3) 4 (4) 6

Sol. [3] $CH_3 - CH = CH - CH(OH) - Me$

Total number of stereo isomers $= 2^n$

$n = 2 \qquad \therefore$ no. of stereo isomers $= 4$

54. The IUPAC name of neopentane is :

(1) 2 - methylbutane (2) 2, 2- dimethylpropane

(3) 2 - methylpropane (4) 2, 2 - dimethylbutane

Sol. [2]

CH_3

1 2 3

$CH_3 - C - CH_3$ 2, 2 - Dimethyl propane

CH_3

55. The set representing the correct order of ionic radius is :

(1) $Li^+ > Be^{2+} > Na^+ > Mg^{2+}$

(2) $Na^+ > Li > Mg^{2+} > Be^{2+}$

(3) $Li^+ > Na^+ > Mg^{2+} > Be^{2+}$

(4) $Mg^{2+} > Be^{2+} > Li^+ > Na^+$

Sol. [2]

Based on effective nuclear charge, the correct order of ionic radius is $Na^+ > Li > Mg^{2+} > Be^{2+}$

56. The two functional groups present in a typical carbohydrate are :

(1) –OH and – COOH (2) –CHO and – COOH

(3) > C = O and – OH (4) –OH and – CHO

Sol. [3] > C = O and – OH carbonyl group & alcohol group present

57. The bond dissociation energy of B-F in BF_3 is 646 kJ mol^{-1} whereas that of C - F in CF_4 is 515 kJ mol^{-1} . The correcxt reason for higher B-F bond dissociation energy as compared to that of C - F is:

(1) smaller size of B-atom as compared to that of C - atom

(2) stronger σ bond between B and F in BF_3 as compared to that between C and F in CF_4

(3) significant $p\pi - p\pi$ interaction between B and F in BF_3 whereas there is no possibility of such interaction between C ad F in CF_4

(4) lower degree of $p\pi - p\pi$ interaction between B and F in BF_3 than that between C and F in CF_4

Sol. [3] $p^5 - p^x$ back bonding decreases the bond length in BF_3 and hence increases the bond energy.

58. In Cannizzaro reaction given below :

$$2\,Ph\,CHO \xrightarrow{\overset{\ominus}{O}H} Ph\,CH_2OH + PhC\overset{\ominus}{O}_2$$

the slowest step is :

(1) the attack of : $\overset{\ominus}{O}H$ at the carboxyl group

(2) the transfer of hydride to the carbonyl group

(3) the abstraction of proton from the carboxylic group

(4) the deprotonation of $Ph\,CH_2OH$

Sol. [2] The transfer of hydride to the carbonyl group is RDS

59. Which of the following pairs represents linkage isomers ?

(1) $[Cu(NH_3)_4][PtCl_4]$ and $[Pt(NH_3)_4][CuCl_4]$

(2) $[Pd(PPh_3)_2][(NCS)_2]$ and $[Pd(PPh_3)_2][(SCN)_2]$

(3) $[CO(NH_3)_5NO_3]SO_4$ and $[CO(NH_3)_5SO_4]NO_3$

(4) $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$

Sol. [2] $\ddot{N}CS$ is an ambidentate ligand

60. Buna-N synthetic rubber is a copolymer of :

(1) $H_2C = CH - \underset{|}{C} = CH_2$ and $H_2C = CH - CH = CH_2$

(2) $H_2C = CH - CH = CH_2$ and $H_5C_6 - CH = CH_2$

(3) $H_2C = CH - CN$ and $H_2C = CH - CH = CH_2$

(4) $H_2C = CH - CN$ and $H_2C = CH - \underset{|}{C} = CH_2$ ($\underset{|}{C}$: CH_3)

Sol. [3] Buna-N synthetic rubber

$H_2C = CH - CN$ and

$H_2C = CH - CH = CH_2$

61. Let a, b, c be such that $b(a + c) \neq 0$. If

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$$, then the value of

n is :

(1) zero (2) any even integer

(3) any odd integer (4) any integer

Sol. (3) Transpose the first determinant we get $\begin{vmatrix} a & -b & c \\ a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \end{vmatrix}$ and

then shift the first row to third row, we get

$$\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a+(-1)^{n+2}a & -b+(-1)^{n+1}b & c+(-1)^n c \end{vmatrix}$$

gives $a + (-1)^{n+2} a = -b + (-1)^{n+1} b = c + (-1)^n c = 0$ only when n = odd integer.

62. If the mean deviation of the numbers 1, $1 + d$, $1 + 2d$, ……, $1 + 100d$ from their mean is 255, then the d is equal to

(1) 10.0 (2) 20.0 (3) 10.1 (4) 20.2

Sol. (3)

Mean of given data $= \frac{1+(1+d)+(1+2d)+.....+(1+100d)}{101} = 1 + 50d$

Given M.D. $= 255 \Rightarrow \frac{|-50d|+|49d|+.....+.....+|50d|}{101} = 255$

$2d\,(1 + 2 + 3 + ... + 50) = 255 \times 101 \qquad d = 10.1$

63. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is

(1) greater than 4ab (2) less than 4b

(3) greater than – 4ab (4) less than –4ab

Sol. (3)

Discriminant of the 1st equation $c^2 - 4ab < 0 \Rightarrow c^2 < 4ab$ …..(1)

Then the value of 2nd expression $= \frac{-D}{4A} = -c^2$ …….(2)

From (1) & (2) given expression > 4ab.

64. Let **A** and **B** denote the statements

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$; **B** : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then :

(1) A is true and B is false (2) A is false and B is true

(3) both A and B are true (4) both A and B are false

Sol. (3)

$2\,(\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)) + 3 = 0$

$$2\sum_{\text{exp and}}\cos(\alpha - \beta) + \left(\cos^2\alpha + \sin^2\alpha\right) + \left(\cos^2\beta + \sin^2\beta\right) + \left(\cos^2\gamma + \sin^2\gamma\right) = 0$$

$$\Rightarrow \left(\sum\cos\alpha\right)^2 + \left(\sum\sin\alpha\right)^2 = 0 \Rightarrow \begin{matrix}\sum\cos\alpha = 0 \\ \sum\sin\alpha = 0\end{matrix}$$

65. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for

(1) no value of p (2) exactly one value of p

(3) exactly two values of p (4) more than two values of p

Sol. (2) Let m be the slope of common line then by condition of perpendicularity.

$\frac{-P(P^2+1)}{-1} m = -1$ …….(1) and $(P^2 + 1)\, m = 1$…….(2)

Divide (1) by (2) $\Rightarrow$ P = –1

66. If A, B and C are three sets such that

$A \cap B = A \cap C$ and $A \cup B = A \cup C$, then :

(1) A = B (2) A = C (3) B = C (4) $A \cap B = \phi$

Sol. (3) Check the Answer B = C is correct.

67. If $\bar{u}, \bar{v}, \bar{w}$ are non-coplanar vectors and p, q are real numbers, then the equality

$\left[3\bar{u}\ p\bar{v}\ p\bar{w}\right] - \left[p\bar{v}\ \bar{w}\ q\bar{u}\right] - \left[2\bar{w}\ q\bar{v}\ q\bar{u}\right] = 0$ holds for :

(1) exactly one value of (p, q)

(2) exactly two values of (p, q)

(3) more than two but not all values of (p, q)

(4) all values of (p, q)

Sol. (1) Here $\left[\bar{u}\ \bar{v}\ \bar{w}\right] \neq 0$

$\left[3\bar{u}\ p\bar{v}\ p\bar{w}\right] - \left[p\bar{v}\ \bar{w}\ q\bar{u}\right] - \left[2\bar{w}\ q\bar{v}\ q\bar{u}\right] = \left[3p^2 + 2q^2 - pq\right]\left[\bar{u}\ \bar{v}\ \bar{w}\right]$

gives $3p^2 + 2q^2 - pq = 0$ which is possible only when $p = q = 0$

68. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals :

(1) (6, –17) (2) (–6, 7) (3) (5, –15) (4) (–5, 5)

Sol. (2) Point (2, 1, –2) will satisfy the plane $x + 3y - \alpha z + \beta = 0$,

$\therefore\ 2 + 3.(1) - 2.(-\alpha) + \beta = 0 \Rightarrow 2\alpha + \beta = -5$ …..(1)

and $3 - 15 - 2\alpha = 0 \Rightarrow \alpha = -6$

putting value of α in equation (1) $\Rightarrow \beta = 7$

69. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is :

(1) less than 500 (2) at least 500 but less than 750

(3) at least 750 but less than 1000 (4) at least 1000

Sol. (4) No. of ways of selecting 4 novel from $6 = {}^6C_4$

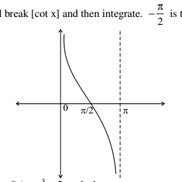
And 1 dictionary from 3 dictionary $= {}^3C_1$

$\therefore$ total arrangement $= [4.{}^6C_4\ {}^3C_1] \geq 1000$.

70. $\int_0^\pi [\cot x]dx$, where [.] denote the greatest integer function, is equal to :

(1) $\frac{\pi}{2}$ (2) 1 (3) –1 (4) $-\frac{\pi}{2}$

Sol. (4) We will break [cot x] and then integrate. $-\frac{\pi}{2}$ is the solution.



71. For real x, let $f(x) = x^3 + 5x + 1$, then

(1) f is one-one but not onto R (2) f is onto R but not one-one

(3) f is one-one and onto R (4) f is neither one-one nor onto R

Sol. (3) If $f(x)$ is odd degree polynomial then f(x) will onto function

$f(x) = x^3 + 5x + 1 \Rightarrow f'(x) = 3x^2 + 5 > 0 \Rightarrow f(x) \uparrow$ Inc.

then one – one function

72. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than :

(1) $\frac{1}{\log_{10}4 - \log_{10}3}$ (2) $\frac{1}{\log_{10}4 + \log_{10}3}$

(3) $\frac{9}{\log_{10}4 - \log_{10}3}$ (4) $\frac{4}{\log_{10}4 - \log_{10}3}$

Sol. (1) p= 1/4, q = ¾

$P(x \geq 1) \geq \frac{9}{10} \Rightarrow 1 - P(x = 0) \geq \frac{9}{10} \Rightarrow {}^nC_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^n \leq \frac{1}{10}$

$\left(\frac{3}{4}\right)^n \leq \frac{1}{10}$ take log and solve to find n.

73. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1, 1) for :

(1) all values of p (2) all except one value of p

(3) all except two vales of p (4) exactly one value of p

Sol. (2) $S_1 + \lambda S_2 = 0$

$(x^2 + y^2 + 3x + 7y + 2p - 5) + \lambda\,(x^2 + y^2 + 2x + 2y - p^2) = 0$ ……(1)

Put (1, 1) then $\lambda = \frac{p^2 - 6}{2p + 7} \neq -1, \quad \forall\, p \in R - \{-1\}$

74. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are :

(1) 6, –3, 2 (2) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (3) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$ (4) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$

Sol. (3) Projection on given coordinate axes x, y & z are respectively $lL = 6$, $mL = -3$, $nL = 2$ Sq. & adding

$L^2\left(l^2 + m^2 + n^2\right) = 36 + 9 + 4$; $L = 7 \Rightarrow l = \frac{6}{7}, m = \frac{-3}{7}, n = \frac{2}{7}$

75. If $\left|Z - \frac{4}{z}\right| = 2$, then the maximum value of $|Z|$ is equal to :

(1) $\sqrt{3} + 1$ (2) $\sqrt{5} + 1$ (3) 2 (4) $2 + \sqrt{2}$

Sol. (2) Using triangle inequality.

$|z_1| - |z_2| \leq |z_1 + z_2| \leq |z_1| + |z_2|$

Let $|z_1| = |z|$ & $|z_2| = \left|\frac{-4}{z}\right| = \frac{4}{|z|}$ and let $|z| = r \Rightarrow \left|r - \frac{4}{r}\right| \leq z$

$\Rightarrow r \in \left[\sqrt{5} - 1, \sqrt{5} + 1\right]$

76. Three distinct points A, B and C are given in the 2 – dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then the circumcenter of the triangle ABC is at the point :

(1) (0, 0) (2) $\left(\frac{5}{4}, 0\right)$ (3) $\left(\frac{5}{2}, 0\right)$ (4) $\left(\frac{5}{3}, 0\right)$

Sol. (2) Given $3PA = PB \Rightarrow \sqrt{(x+1)^2 + y^2} = 3\sqrt{(x-1)^2 + y^2}$

$8x^2 + 8y^2 - 20x + 8 = 0$ then circumcenter $\left(\frac{5}{4}, 0\right)$

77. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is :

(1) 0 (2) 2 (3) 7 (4) 8

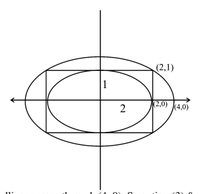
Sol. (2) When 8^{2n} is divided by 9, remainder is 1 because power is even when $(62)^{2n+1}$ is divided by 9 the remainder is 8 because the power is odd. So, remainder is $1 - 8 = -7$. but the remainder can't be –ve

$\therefore$ the actual remainder is $= 9 - 7 = 2$.

78. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is :

(1) $x^2 + 16y^2 = 16$ (2) $x^2 + 12y^2 = 16$ (3) $4x^2 + 48y^2 = 48$ (4) $4x^2 + 64y^2 = 48$

Sol. (2)



The required ellipse passes through (4, 0). So option (3) & (4) are wrong.

From the diagram, the required ellipse passes through (2, 1) also.

79. The sum of infinity of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...$ is :

(1) 2 (2) 3 (3) 4 (4) 6

Sol. (2) Let $S = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...$ Which is AGP.

$S_\infty = \frac{2/3}{1 - 1/3} + \frac{4/9}{\left(1 - \frac{1}{3}\right)^2} = 2$

$\therefore$ sum of infinite terms of given series $= 1 + 2 = 3$

80. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1 and c_2 are arbitrary constants, is

(1) $y' = y^2$ (2) $y'' = y'\,y$ (3) $yy'' = y'$ (4) $yy'' = (y')^2$

Sol. (4) Taking log on both the sides and then differentiate twice.

81. One ticket is selected at random from 50 tickets numbered 00, 01, 02, …., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals :

(1) $\frac{1}{14}$ (2) $\frac{1}{7}$ (3) $\frac{5}{14}$ (4) $\frac{1}{50}$

Sol. (1) Sample space will be 14 = [(00), (01)……(09), (10), (20), (30), (40)]

Possible favourable cases = 1 = (08)

Probability = favourable case/sample space = 1/14

82. Let y be an implicit of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then y' (1) equals :

(1) –1 (2) 1 (3) log 2 (4) – log 2

Sol. (1) $\frac{dy}{dx} = -\frac{\text{(differentiate w.r.t. x as y constant)}}{\text{(differentiate w.r.t. y as x constant)}}$

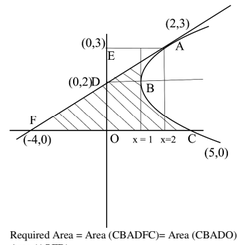
$\frac{dy}{dx} = -\left(\frac{2\times x^{2x}(1 + \ln x) - 2x^x(1 + \ln x).\cot y}{2x^x \operatorname{cosec}^2 y}\right)$

$\left.\frac{dy}{dx}\right|_{\text{at } x=1} = -\left(\frac{1 - \cot y}{\operatorname{cosec}^2 y}\right)$ (given y (1) = 0) = –1

83. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent of the parabola at the point (2, 3) and the x-axis is :

(1) 3 (2) 6 (3) 9 (2) 12

Sol. (3)



Required Area = Area (CBADFC)= Area (CBADO) –Area (ΔADE) + Area (ΔOFD)

$\Rightarrow \int_0^3 \left[(y-2)^2 + 1\right]dy - \frac{1}{2}(1)\times(2) + \frac{1}{2}\times 4\times 2 = 9$

84. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that x = 0 is the only real root of P' (x) = 0. If P(–1) < P(1), then in the interval [–1, 1] :

(1) P(–1) is the minimum and P(1) is the maximum of P

(2) P(–1) is not minimum but P(1) is the maximum of P

(3) P(–1) is the minimum but P(1) is not the maximum of P

(4) neither P(–1) is the minimum nor P(1) is the maximum of P

Sol. (2) $P'(x) = 4x^3 + 3x^2 + 2bx + c$; at x = 0, c = 0

$p'(x) = 4x^3 + 3ax^2 + 2bx \qquad p'(x) = x\left(\underset{+ve}{4x^2 + 3ax + 2b}\right)$

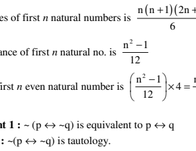
then, [–1, 0], p'(x) < 0 $\downarrow$ Dec. (p(–1) is max imum)

[0, 1], p'(x) > 0 $\uparrow$ Inc (p(1) is max imum)

85. The shortest distance between the line y – x = 1 and the curve $x = y^2$ is :

(1) $\frac{3\sqrt{2}}{8}$ (2) $\frac{2\sqrt{3}}{8}$ (3) $\frac{3\sqrt{2}}{5}$ (4) $\frac{\sqrt{3}}{4}$

Sol. (1)



Differentiating equation of curve1 = 2y. $\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2(\alpha)}$

Slope of given line is 1 then $\frac{1}{2\alpha} = 1 \Rightarrow \alpha = 1/2$

Point on curve (α^2, α) i.e $\left(\frac{1}{4}, \frac{1}{2}\right)$

S.T.= $\left|\frac{\frac{1}{2} - \frac{1}{4} - 1}{\sqrt{2}}\right| = \frac{3\sqrt{2}}{8}$

Directions : Questions number 86 to 90 are Assertion – Reason type questions. Each of these questions contains two statements :

Statement -1 (Assertion) and **Statement – 2 (Reason)**

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(1) Statement -1 is true, Statement -2 is true;

Statement-2 is the correct explanation for Statement-1

(2) Statement – 1 is true, Statement – 2 is true;

Statement – 2 is not a correct explanation for Statement – 1

(3) Statement – 1 is true, Statement – 2 is false.

(4) Statement -1 is false, Statement – 2 is true.

86. Let $f(x) = (x + 1)^2 - 1$, $x \geq -1$.

Statement 1 : The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$

Statement 2 : f is a bijection

Sol. (1) $f^{-1}(x) - \sqrt{(x+1)} - 1 \Rightarrow$ Assertion is true. f^{-1} exist.

But $f^{-1}(x) = f(x) \Rightarrow (x + 1)^2 - 1 = \sqrt{(x+1)} - 1$; x = –1, 0

$\therefore$ For reason {x : x = 0, –1} & f is bijection. Because f^{-1} exist.

87. Let $f(x) = x\,|\,x\,|$ and $g(x) = \sin x$.

Statement 1 : gof is differentiable at x = 0 and its derivative is continuous at that point.

Statement 2 : *gof* is twice differentiable at x = 0

Sol. (1) gof (x) = sin (x |x|) = $\begin{cases}\sin x^2 \text{ if } x \geq 0 \\ -\sin x^2 \text{ if } x < 0\end{cases}$

Here gof is twice differentiable at x = 0, Also gof is differentiable and continuous at x = 0.

88. Statement 1 : The variance of first n even natural numbers is $\frac{n^2 - 1}{4}$.

Statement 2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$.

Sol. (4) Variance of first n natural no. is $\frac{n^2 - 1}{12}$

Variance of first n even natural number is $\left(\frac{n^2 - 1}{12}\right)\times 4 = \frac{n^2 - 1}{3}$

89. Statement 1 : ~ (p ↔ ~q) is equivalent to p ↔ q

Statement 2 : ~(p ↔ ~q) is tautology.

Sol. (3)

p	q	~q	p ↔ q	P ↔ ~q	~ (p ↔ ~q)
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	F	T

90. Let A be a 2 × 2 matrix

Statement 1 : adj (adj A) = A

Statement 2 : $|\text{adj}\,A| = |A|$

Sol. (1)

adj. (adj A) = $|A|^{n-2}$ A = A (n = 2)

$|\text{adj}\ A| = |A|^{n-1} = |A|$